

Multivariate Nonparametric Methods Based on Spatial Signs and Ranks

Hannu OJA
Tampere School of Public Health, Finland

Abstract: Classical multivariate statistical inference methods (Hotelling's tests, multivariate analysis of variance, multivariate regression, etc.) are based on the use of L_2 criterion functions. In this talk we consider alternative L_1 criterion functions with related tests and estimates. Multivariate L_1 criterion functions for estimation problem are used to extend the concepts of sign and rank to the multivariate case.

We consider three different multivariate L_1 criterion functions utilizing Euclidean distance. The first criterion function, the mean deviation of the multivariate residuals, is the basis for the so called least absolute deviation (LAD) methods; it yields different spatial median-type estimates and spatial sign tests in the one-sample, two-sample, c -sample and finally general linear regression settings. The second objective function is the mean difference of the residuals, and the third one is the sum of of the lengths of the pairwise sums and pairwise differences of the residuals. The second and third objective functions generate multivariate Hodges-Lehmann type estimates and spatial rank and signed-rank tests for different location problems. For these ideas, see also Hettmansperger and Aubuchon (1988).

In the talk, we briefly review the theory of the multivariate spatial sign and rank methods, tests and estimates, in the one sample, several samples and, finally, multivariate linear regression cases. See Mottonen and Oja (1995) and Oja and Randles (2004). Transformation-retransformation technique is used to obtain affine invariant tests and equivariant estimates. See e.g. Hettmansperger and Randles (2003). The theory is illustrated with examples.