Rank-based tests of multivariate independence in independent component models

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Abstract: The so-called independent component (IC) model states that the observed p-vector X is generated via $X = \Lambda Z + \mu$, where μ is a p-vector, Λ is a full-rank matrix, and the centered random vector Z has independent marginals Z_i . We consider the problem of testing, on the basis of n i.i.d. copies of $X = (X^{(1)'}, X^{(2)'})'$, the null hypothesis under which the multivariate marginals $X^{(1)}$ and $X^{(2)}$ are independent. Under a symmetry assumption on the Z_i's, we propose a class of semiparametric procedures, which are based on the componentwise signed ranks of the estimated independent components (the latter are obtained under the null via a recent procedure due to Oja et al. 2006). This componentwise signs-and-ranks methodology was rst proposed by Puri and Sen (1971). However, unlike the Puri and Sen tests, our tests (i) are affine-invariant and (ii) achieve, for adequately chosen scores, local and asymptotic optimality (in the Le Cam sense) at given densities. They are also valid without any moment assumptions. Local powers and asymptotic relative efficiencies with respect to the classical Gaussian procedure (namely, Wilks Gaussian LRT test) are derived. Finite-sample properties are investigated through a Monte- Carlo study.